# Computing Price-Cost Margins in a Durable Goods Environment

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Preliminary

- We typically cannot observe marginal cost.
- Methods for computing marginal cost from demand and equilibrium assumptions are popular.
- How should we proceed in a dynamic environment?
- In particular, digital camcorders, which are durable.

# Why estimate marginal costs for durable goods?

- To understand market power in the industry.
  - Can use cost estimates to compute price-cost margins.
- To evaluate when innovation is occurring in this sector.
  - Useful to understand if competition or concentration is causing cost reductions.
- To understand the extent to which forward-looking firm behavior matters.
  - For instance, smaller firms may mostly cannibalize other firm sales.
- Run counterfactual experiments, e.g. merger simulations.
  - Long-term goal: Why do prices fall in this industry?

- Estimate demand.
- Impose equilibrium assumption.
- Compute marginal revenue.
  - If price-setting, we need to invert to get MR wrt Q.
- Theory says this is marginal cost.

- Derivative of marginal revenue is dynamic.
- It incorporates the change in current market share AND the change in the future stream of profits.

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- It incorporates the change in current market share AND the change in the future stream of profits.
- In a durable goods framework:
  - a lower price today steals consumers from the future AND
  - affects future pricing decision.

Compute marginal revenue in a way that resolves the effect of pricing today on:

- Today's market share.
- Future consumer demand.
- Dynamic strategic interactions.

# Problems with dynamics.

- Computational: Large state space.
- Provide the second state of the second stat
  - We borrow from Berry & Pakes (2000) and Bajari, Benkard & Levin (2005) address these issues.
  - However, BP and BBL rely on do not derive the value of marginal cost that rationalizes each price.
    - Loosely, they identify the parameters in the marginal cost function, but not the error term.
    - If the error term is a large component of marginal cost, our analysis of marginal cost will be erroneous.

- Finding MC for each price associated with Bresnahan (1987) and Berry, Levinsohn & Pakes (1995).
- Auction equivalent: Gurre, Perrigne & Vuong.
- Closest may be Pesendorfer & Jofre-Bonet (2003) in an auction framework (with different goals).
- Other papers that find MC in a dynamic framework:
  - Estaban & Shum (2007), Goettler & Gordon (2011), Kim (2014).

Step 1 Estimate reduced-form approximation of pricing strategy. (BBL)

Step 2 Construct dynamic FOC and invert to compute MC. (BLP)

- Assume there is a final period, and proceed by backwards induction.
- In each period, compute current market share and expected future profits.
  - Use Step 1 to predict prices in the future.
  - But we use structural transitions from our demand model.
- Change one price by 5%, and recompute.
- Compute MR from price change.
- Invert to obtain marginal cost.

# Price by time, "Big 4" firms.



- Marginal costs for firms are lower when we include dynamics!
- Even lower for firms with large market shares.
  - Dynamics significant in preventing Sony (60%+) from lowering prices early on.
- Price-cost margins fall over time.
- High value products have higher price-cost margins.
- More work to do ...

- Mass of consumers M.
- Discrete time, live forever.
- Consumers make a discrete choice what to buy or to wait each period.
- Product is infinitely durable.
- Consumers hold one good at a time.
- Consumer holdings described by *H*<sub>t</sub>.

- Share to *j* in *t* is  $s_{jt}(P_t, H_t)$ .
  - *P<sub>t</sub>* is vector of prices.
- Consumer holdings evolve:

$$H_{t+1} = g_1(H_t, S_t, P_t, \Omega_t^c)$$

•  $\Omega_t^c$  are state variables for consumer.

- Firms are indexed by  $f = 1, \ldots, F$ .
- Each period, there are  $J_t$  products available.
- Firm *f* produces all  $j \in \mathfrak{F}_{ft}$ .
- Each product has a mean flow utility and a marginal cost mc<sub>jt</sub>.
- Product utilities (past, present and future) are known and exogenous.
- Firms know all current marginal costs but have uncertainty over all future marginal costs.
  - Information is symmetric across firms.
- Firm picks price  $p_{jt}$  for all  $j \in \mathfrak{F}_{ft}$ .

- State space for firms:  $\widehat{\Omega}_t$ .
- Transitions are Markov:  $\widehat{\Omega}_{t+1} = g_2(P_t, \widehat{\Omega}_t)$ .
- Value function:

$$V(\widehat{\Omega}_{ft}) = \max_{P_{ft}} E\left[ \sum_{\tau=t}^{\infty} \sum_{j \in \mathfrak{F}_{f\tau}} \left( p_{j\tau} - mc_{j\tau} \right) Ms_{j\tau}(S_{\tau}, P_{\tau}) \middle| \widehat{\Omega}_{ft} \right].$$

- Markov Perfect Equilibrium.
- First-order condition for price *jt*:

$$\begin{split} s_{jt}(S_t, P_t) + \sum_{k \in \mathfrak{F}_{ft}} \left( p_{kt} - mc_{kt} \right) \frac{\partial s_{kt}(S_t, P_t)}{\partial p_{jt}} \\ + \beta \frac{\partial}{\partial p_{jt}} E\left[ V_f(\widehat{\Omega}_{ft+1}) | \widehat{\Omega}_{ft}, P_t \right] = 0. \end{split}$$

• Plan: Use this equation to compute marginal cost.

# Consumer demand, with detail.

We follow Gowrisankaran & Rysman (2012) exactly.

- The mean flow utility of product *j* in period *t* to consumer *i* is δ<sup>f</sup><sub>ijt</sub>.
- Flow utility in period of purchase:

$$u_{ijt} = \delta^{f}_{ijt} - \alpha_{i} \ln(p_{jt}) + \varepsilon_{ijt}.$$

•  $\varepsilon_{ijt}$  is iid EV.

• 
$$\delta_{ijt}^f = \overline{\delta}_{jt}^f + \sigma_1 \nu_{i1}$$
.

- $\alpha_i = \alpha + \sigma_2 \nu_{i2}$ .
- $\nu_{i1}$ ,  $\nu_{i2}$  distributed  $\mathcal{N}(0, 1)$
- $\alpha$ ,  $\sigma_1$  and  $\sigma_2$  are to be estimated.

- Consumers track flow utility of the product they own: δ<sup>0</sup><sub>it</sub>.
- $\delta_{it}^0 = 0$  for t = 1, up until time of first purchase.
- $\delta_{it}^0 = \delta_{jt}^f$  for *t* after purchase.

# Modeling purchase options.

- The logit inclusive value (δ<sub>it</sub>) captures the value of purchase.
- The inclusive value:

$$\delta_{it} = \ln \sum_{j \in J_t} \exp\left(\delta_{ijt}^f + \beta E\left[\left.V_i^c(\delta_{ijt}^f, \delta_{it+1})\right| \delta_{it}\right]\right)$$

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#### Assumption: Inclusive Value Sufficiency (IVS):

 $P(\delta_{it+1}|\Omega_t^c) = P(\delta_{it+1}|\delta_{it}).$ 

- Consumers predicts the future distribution of δ<sub>it</sub> based only on current δ<sub>it</sub>, rather than all of Ω<sup>c</sup>.
- Reduces the state space!

• We implement IVS with an AR1 function form:

$$\delta_{it+1} = \gamma_{0i} + \gamma_{1i}\delta_{it} + \eta_{it}$$

- We estimate these parameters from  $\delta_{it}$  in our model.
  - Thus, we impose rational expectations.
- The parameters  $\{\gamma_{0i}, \gamma_{1i}, \sigma_i^{\nu}\}$  represent future expectations.

## Consumer value function.

- IVS provides two important simplifications:
  - Value function depends on two scalars ( $\delta_{it}^0$  and  $\delta_{it}$ ).
  - Reduced-form approximation of the supply side means we can estimate demand separately from supply.
- Consumer value function:

$$V_i^c(\delta_i^0, \delta_i) = \ln\left(\exp(\delta_i) + \exp\left(\delta_i^0 + \beta E\left[\left.V_i^c(\delta_i^0, \delta_i')\right|\delta_i\right]\right)\right).$$

 For a given vector of mean utilities, we solve for Bellman, AR1, and IV simultaneously.

- Consumer solution implies market shares for each type *i* and period *t*.
- We aggregate over types to get a predicted market share  $\hat{s}_{jt}$ .
- We use a BLP fixed point equation to solve for mean utilities:

$$\overline{\delta}_{jt}^{f} = \overline{\delta}_{jt}^{f} + \ln(s_{jt}^{*}) - \ln(\widehat{s}_{jt}^{*})$$

#### Solve simultaneously for:

- $\delta_{it}$  Logit inclusive value.
- $\{\gamma_{0i}, \gamma_{1i}, \sigma_i^{\eta}\}$  AR1 approximation of expectations.
  - $V_i^c$  Value function from Bellman.

 $\overline{\delta}_{it}^{t}$  Mean flow utilities.

- Use simulation over *i*.
  - For elements indexed by *i*, we must solve separately for each draw *i*.

# Consumer problem overview.

- Demand is a random-coefficient logit model with RCs on the constant term and price only.
- Consumers hold 1 good at a time.
- Product is infinitely durable.
- Consumers can update their product today or wait.
- Consumers have rational expectations about the future evolution of offerings, based on a reduced-form approximation of how the supply side evolves.

#### Implication:

More sales today imply lower sales the next period.

- Let  $\Omega_t$  equal  $\widehat{\Omega}_t$  but for the marginal costs.
  - $\Omega_t$  consists of the state variables that are observable to the econometrician.
- Let  $P_t = \psi(\Omega_t, U_t)$ .
  - *U<sub>t</sub>* is the vector of random draws for all products in *t*.
  - Distribution of *U<sub>t</sub>* is related to distribution of *MC<sub>t</sub>*.
- Step 1: Specify functional form for  $\psi$  and estimate.

- There is a final period *T*, past what we observe in the data.
  - In practice, we assume product offerings stay the same as the last period in the data.
- Draw *ns* values of  $U_t^s$ ,  $s = 1, \ldots, ns$ .
  - Distribution is based on results of first-step estimation.
- Compute the distribution of prices for each product and period.

$$P_t^s = \psi(\Omega_t, U_t^s).$$

#### Last period

• FOC in last period:

$$s_{jT}(P_T^s, H_T) + \sum_{k \in \mathfrak{F}_{fT}} (p_{kT}^s - mc_{kT}^s) \frac{\partial s_{kT}(P_T^s, H_T)}{\partial p_{jT}^s} = 0.$$

Matrix notation:

$$P_{fT}^{s} + \Lambda_{fT}^{s,-1} S_{fT}(P_{fT}^{s}, H_{t}) = MC_{fT}^{s}.$$

- $\Lambda_{ft}$  is the matrix  $\partial S_{ft} / \partial P_{ft}$ .
  - Note:  $\Lambda_{ft}$  is for one firm. All elements are non-zero.
- We obtain a <u>distribution</u> of marginal costs in the last period.

# Constructing $\Lambda_{ft}$ .

- For a given  $P_T^s$ , compute  $S_{fT}(P_T^s, H_t)$ .
- Change one price by a small discrete amount (5%).
  - Call new vector P<sup>s'</sup><sub>T</sub>.
- Compute  $S_{fT}(P_T^{s\prime}, H_t)$ .
- Use discrete approximation to derivative to construct Λ<sub>ft</sub>.
- That is, element [k, j] is:

$$\Lambda_{ft}[k,j] = \frac{\Delta s_{kT}}{\Delta p_{jT}^{s}}.$$

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Use GR to compute market shares even in *T*.
dynamic demand.

# Other periods

• FOC in matrix notation:

$$P_{fT-1}^{s} + \Lambda_{fT-1}^{s,-1} \left( S_{fT-1}(P_{fT-1}^{s}, H_{T-1}) + \beta \frac{\partial}{\partial P_{fT-1}^{s}} EV \right) = MC_{fT-1}^{s}.$$
  
•  $EV = E \left[ V_{f}(\Omega_{fT}) | \Omega_{fT-1}^{s}, P_{T-1}^{s} \right] =$   
 $E \left[ V_{f}(\Omega_{ft}) | \Omega_{ft-1}^{s}, P_{t-1}^{s} \right] = \frac{1}{ns} \sum_{\tau=t}^{T} \sum_{m=1}^{ns} \sum_{k \in \mathfrak{F}_{f\tau}} (p_{k\tau}^{ms} - mc_{k\tau}^{m}) s_{k\tau}(P_{\tau}^{ms}, H_{\tau}^{s})$ 

.

# Computing

How to compute:

$$E\left[V_f(\Omega_{ft})|\Omega_{ft-1}^s, P_{t-1}^s\right] = \frac{1}{ns} \sum_{\tau=t}^T \sum_{m=1}^{ns} \sum_{k \in \mathfrak{F}_{f\tau}} \left(p_{k\tau}^{ms} - mc_{k\tau}^m\right) s_{k\tau} \left(P_{\tau}^{ms}, H_{\tau}^{ms}\right) = \frac{1}{ns} \sum_{k \in \mathfrak{F}_{f\tau}} \left(p_{k\tau}^{ms} - mc_{k\tau}^m\right) s_{k\tau} \left(P_{\tau}^{ms}, H_{\tau}^{ms}\right)$$

- For each shock *m*, *mc<sup>m</sup>* is already computed because we are using backward induction.
- Starting from any state in t, we observe holdings  $H_t$ .
  - Compute *p*<sup>s</sup> from reduced-form equation and shocks.
  - Compute market share  $s_t(p_t^s, H_t)$ .
  - Implies holdings  $H_{t+1}^s$ .
  - Compute prices  $p_{t+1}^{ms}$ .
  - Compute shares  $\vec{s}_{t+1}(p_{t+1}^{ms}, H_{t+1}^s)$ , and thus  $H_{t+2}^{ms}$ .
  - Compute prices  $p_{t+2}^{ms}$  etc.
- Adjust one starting price by 5%, recompute to get derivative.

- We now have the distribution of *MC* in each period.
- We can repeat our solution for *MC*, but this time substitute observed prices for simulated prices.
- Thus, we find the marginal cost that rationalizes all of the observed prices (conditional on the distribution of future *MC* that we have computed).

- Perfect foresight of *MC* allows for elegant solution of all *MC* simultaneously.
- We need  $\wedge$  for all products simultaneously.
- Requires derivative of market share from prices in different periods, and we cannot use simulation to do computation.
- Thus, using simulation changes the solution technique.
- Note, we could also implement asymmetric info in current MC, but that deviates from BLP and is a little harder.

- Sales and average price for digital camcorders.
- Monthly for March 2000 to May 2006.
- Does not account for Walmart or on-line sales.
- NPD Techworld.
- 383 products, 11 brands, 4,436 observations.

## Number of models, "Big 4" firms.



#### Number of models, "Next 3" firms.



#### Market share over time, "Big 4" firms.



Variable	Measurement	Mean	Std.	Min	Max
	Product-level variable				
Product quality	Mean flow utility of product.	0.0008	0.045	-0.12	0.09
	Firm-level variables				
Firm average product quality	Average of mean flow utilities	0.0015	0.022	-0.11	0.06
	excluding product in question.				
Firm size	Number of products firm owns	13.75	6.2	1	31
	Market-level variables				
Market average product quality	Average of mean flow utilities	0.0008	0.006	-0.19	0.2
	of all products in market				
Market size	Number of products in market.	62.56	13.95	27	98
Consumer holdings	Percentage of the population	0.045	0.03	0	0.1
	have purchased the good.				

- Regress price on state variables, but which ones?
- BLP instruments:
  - Product quality.
  - Variables that capture the price-cost margin:
    - Counts of own products and rival products.
    - Average characteristics of own and rival products.
- We use one characteristic: mean utilities  $\overline{\delta}_{it}^{t}$ 
  - If we had random coefficients on more characteristics, we would use those characteristics also.

#### Implement Step 1. Consumer holdings.

- Consumer holdings also predicts prices.
- We use the share of consumers that hold the good.
- Other measures (quality of products they hold, variance across consumer types) had little predictive power.
- Note that in our specification of demand, there is almost no repurchase.

# **Results for Step 1**

	Dependent variable: In(price)				
	Firm Random Effec (1)		Firm Fixed Effect (2)		
Variable	Coefficient	Std.	Coefficient	Std.	
Product quality	6.65***	(0.21)	6.62***	(0.1)	
Firm average product quality	0.56	(0.65)	0.48	(0.33)	
Firm size	0.013***	(0.002)	0.013***	(0.002)	
Market average product quality	-2.8***	(0.95)	-2.66***	(0.94)	
Market size	-0.003***	(0.001)	-0.003***	(0.001)	
Consumer holdings	-9.61***	(0.25)	-9.6***	(0.3)	
Constant	6.68***	(0.06)	6.18***	(0.06)	
Observations	4,436		4,436		
Adjusted R <sup>2</sup>	0.641		0.662		
Residual Std. Error	0.32		0.32		
F statistic	774.27***( <i>df</i> = 16; 4419			<sup>f</sup> = 16; 4419)	



- T = 110 (add 25 periods to data).
- *ns* = 16.
- β = 0.99.
- Discretize state space for  $\delta_{it}$  into 100 values.
- Discretize state space for  $\delta_{it}^0$  into 21 values.

## Average marginal cost by period and firm.



## Average price-cost difference by period and firm.



### Price-cost difference by flow utility.



# Average difference between static and dynamic marginal costs.



- Dynamics affect computation of marginal cost
- Much more so for the biggest firm
- We have more work to do ...